NOTES AND CORRESPONDENCE

A Simple Comparison of Four Physics–Dynamics Coupling Schemes

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1 February 2002 and 27 April 2002

ABSTRACT

Four schemes (referred to here as explicit, implicit, split-implicit, and symmetrized split-implicit) for coupling physics parameterizations to the dynamical core of numerical weather and climate prediction models have been studied in the context of a simplified, canonical model problem. This problem models the dynamics by a representation of the terms responsible for gravitational oscillations and models the physics by both a constant forcing term and a linear damping term, representative of horizontal or vertical diffusion. The schemes have been analyzed in terms of their numerical stability and accuracy. Two of the schemes (the explicit and split-implicit) have been studied previously in the context of a three-time-level discretization. Those results are confirmed here for a two-time-level discretization. The two other schemes (the implicit and the novel symmetrized split-implicit) are both found to be second-order accurate and unconditionally stable, and both represent improvements over the explicit and split-implicit schemes. The symmetrized split-implicit has the additional advantage over the implicit scheme, for simplicity and computational efficiency, of separating the physics and dynamics steps from each other. The canonical problem considered here is a considerable simplification of any real physics–dynamics coupling, which limits the generality of the conclusions drawn. However, such simplification allows detailed analysis of some important aspects and motivates further work on both broadening and deepening understanding of physics–dynamics coupling issues.

1. Introduction

Physics parameterization packages are key elements in the success of numerical weather and climate prediction models. The accuracy and complexity of these schemes continues to increase apace. It is therefore important for the continued improvement of the models that the benefits and disadvantages of the various strategies employed to couple the physics to the inviscid, adiabatic dynamical cores of the models are well understood.

In numerical models a distinction is usually made between fast and slow timescales because of differing stability considerations (Grabowski and Smolarkiewicz 1996; Wedi 1998; Williamson 1999; Teixeira 2000). An explicit time discretization generally has the virtue of simplicity. For a slow timescale process computational efficiency is usually not hindered by an associated stability-limited time step and an $O(\Delta t)$ accurate discretization therefore arguably suffices. However, for a fast timescale process, an explicit time discretization generally unduly limits the time step due to an overly restrictive stability condition. A more costly implicit time discretization is therefore usually adopted. Even so, as illustrated in Caya et al. (1998, hereinafter referred to as CLZ98) and noted in Wedi (1998), while this can address the stability issue, if the resulting discretization is only $O(\Delta t)$ accurate, then the time step may still be unduly limited due to time truncation error. This motivates the use of an $O(\Delta t^2)$-accurate implicit time discretization of such fast processes. In practice this is difficult to accomplish because the additional complexity associated with the introduction of such parameterized processes gives rise to problems in efficiently coupling them with the time-discretized dynamics.

With a particular semi-implicit semi-Lagrangian model in mind, CLZ98 reduced the physics–dynamics coupling issue to its essence and thereby provided useful insight into the problem. They examined steady-state solutions of numerical discretizations of

$$\frac{dF(t)}{dt} + \sigma F(t) = G, \qquad (1.1)$$

where σ (β in CLZ98's notation) and G are constants.

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The term G is a constant forcing, chosen to model a constant diabatic heating in a full model.

The exact solution of (1.1) is

$$F(t) = F_0 e^{-\sigma t} + \frac{G}{\sigma}, \tag{1.2}$$

where F_0 is the initial amplitude of the transient component of the solution and G/σ is the forced response, and a steady-state solution

$$F^{\text{steady}} = \frac{G}{\sigma}, \tag{1.3}$$

exists provided either $F_0 \equiv 0$ or $Re\{\sigma\} > 0$.

If σ is purely real and greater than zero, the σF term represents a damping process (e.g., boundary layer diffusion). If, however, σ is purely imaginary, then σF represents an oscillatory process. CLZ98 considered the term to be *either* a damping term *or* an oscillatory term. It was this latter interpretation that CLZ98 focused attention on, using this term to model the terms responsible for gravitational oscillations. As such the term was discretized semi-implicitly, that is, using centered time averaging.

Here, in order to develop their model further, the canonical equation of CLZ98 is modified to allow different discretizations of both damping and oscillatory processes, and to allow them to occur *simultaneously* and mutually interact. To this end the σF term is decomposed into two terms: an oscillatory $i\alpha F$ term on the left-hand side, to model the semi-implicit discretization of the terms responsible for gravitational and acoustic oscillations; and a $-\beta F$ term on the right-hand side, to model the incorporation of fast parameterizations such as vertical diffusion. This keeps the analysis tractable while resulting in a problem that still does not depend (at least explicitly) on space but only on time, as in CLZ98. Therefore, a revised canonical problem,

$$\frac{dF}{dt} + i\alpha F = G - \beta F, \tag{1.4}$$

is considered where α ($\neq 0$), β (≥ 0), and G are all real and constant.

The exact solution of the revised canonical problem is obtained as the sum of the time-dependent free, or homogeneous, solution, $F^{\text{free}}_{\text{exact}}(t)$, and the forced solution, $F^{\text{forced}}_{\text{exact}}$ (which in this case is a constant):

$$\begin{aligned} F_{\text{exact}}(t) &\equiv F^{\text{free}}_{\text{exact}} + F^{\text{forced}}_{\text{exact}} \\ &= F_0 e^{-(i\alpha+\beta)t} + \frac{G}{i\alpha + \beta}. \end{aligned} \tag{1.5}$$

If either $F_0 \equiv 0$ or $\beta > 0$, then an exact steady-state solution exists, namely,

$$F^{\text{steady}}_{\text{exact}} = F^{\text{forced}}_{\text{exact}} \equiv \frac{G}{i\alpha + \beta}. \tag{1.6}$$

CLZ98 examined three different ways of coupling physical forcings to a three-time-level semi-implicit discretization of the dynamics. Attention herein is however restricted to an analogous two-time-level semi-implicit dynamical discretization. For the first two coupling schemes examined by CLZ98 (the ''semi-implicit traditional'' and ''semi-implicit split'' methods) terms only materialize at times $t \pm \Delta t$ and not at the intermediate time t. Therefore, the odd-time-step solutions are effectively decoupled from the even ones. Hence, these two schemes are formally equivalent to special cases of two (viz, explicit and split-implicit) of the four couplings examined herein. The other coupling they briefly examined was an explicit, rather than semi-implicit, leapfrog scheme. Stability constraints on the time step mean that this would be prohibitively expensive in practice and consequently is not pursued here. Instead, two further coupling strategies are described and analyzed. The first of these, the implicit coupling, is examined with a view to establishing, at least for the canonical problem herein, a coupling that is arguably ideal from the perspectives of stability and accuracy, albeit suboptimal in terms of efficiency. The remaining coupling, symmetrized split-implicit, aims to combine the stability and accuracy properties of the implicit coupling with the efficiency of the less accurate split-implicit one.

In CLZ98, it was pointed out that the split-implicit coupling can lead to a seriously erroneous steady-state response at a large time step in the context of the Caya et al. (1995) semi-implicit semi-Lagrangian model. Paradoxically this finding is at variance with results obtained in the context of the two-time-level semi-implicit semi-Lagrangian models of Chen and Bates (1996) and others. They also suggested that perhaps the split-implicit coupling could adversely affect the accuracy of the transient component of the flow.

This note provides a possible explanation for this paradox, gives a modest insight into the numerics of four physics–dynamics couplings, considers the accuracy of the transient component of the flow, and introduces the symmetrized split-implicit scheme.

2. Coupling discretizations

In the schemes considered here, the ''dynamics'' of the canonical problem, that is, the left-hand side of (1.4), is always discretized using a two-time-level, semi-implicit scheme. Therefore, the names of the four different coupling schemes refer to how the ''physics,'' that is, the right-hand side of (1.4), is discretized. Each of these couplings is detailed below.

a. *Explicit*

CLZ98's explicit coupling [their Eq. (9)], when applied to the revised canonical equation (1.4), results in

$$\frac{F(t+\Delta t)-F(t)}{\Delta t}+\frac{i\alpha}{2}[F(t+\Delta t)+F(t)] = G-\beta F(t). \tag{2.1}$$

The $-\beta F$ term on the right-hand side represents a diffusive process associated with the physics and has therefore been time discretized in an explicit manner in the spirit of the $P[\Psi(t-\Delta t)]$ term of Eq. (2) of CLZ98.

b. Implicit

Both the stability and accuracy problems of the explicit coupling can in principle (see section 3) be addressed, albeit at non-negligible cost in the full-model context, by the implicit coupling

$$\frac{F(t+\Delta t)-F(t)}{\Delta t}+\frac{i\alpha}{2}[F(t+\Delta t)+F(t)] = G-\frac{\beta}{2}[F(t+\Delta t)+F(t)]. \tag{2.2}$$

Note that for the constant forcing considered by CLZ98 (i.e., $\beta \equiv 0$), this coupling is equivalent to the explicit coupling of (2.1).

c. Split implicit

Applying a split-implicit coupling to discretize (1.4) gives the two-step discretization

$$\frac{F^*-F(t)}{\Delta t}+\frac{i\alpha}{2}[F^*+F(t)] = 0, \tag{2.3}$$

$$\frac{F(t+\Delta t)-F^*}{\Delta t} = G-\beta F(t+\Delta t). \tag{2.4}$$

Both steps are implicit and the second step, in the context of vertical diffusion, can be accomplished by solving a set of tridiagonal problems in the vertical. The first step is a dynamics-only predictor while the second is a physics-only corrector. This conveniently keeps the physics discretization distinct from that of the dynamics. Note that since the $-\beta F$ term is considered to represent a fast process, such as vertical diffusion, the term has been discretized implicitly (i.e., as $-\beta F(t+\Delta t)$) in line, for example, with the European Centre for Medium-Range Weather Forecasts (ECMWF) model (Teixeira 2000).

Eliminating F^* from (2.3)–(2.4) yields the equivalent coupling equation

$$\frac{F(t+\Delta t)-F(t)}{\Delta t}+\frac{i\alpha}{2}[(1+\beta\Delta t)F(t+\Delta t)+F(t)] = \left(1+\frac{i\alpha\Delta t}{2}\right)G-\beta F(t+\Delta t). \tag{2.5}$$

When $\beta \equiv 0$, and after rescaling Δt by a factor of 2 and making allowance for differences in notation, (2.5) is equivalent to the simpler (14) of CLZ98. When $\beta > 0$, an implicit time bias is introduced into the semi-implicitly treated $i\alpha F$ term, which (see section 3) adversely affects accuracy; this is particularly serious for strong damping, that is, large $\beta\Delta t$.

d. Symmetrized split implicit

As will be shown in sections 3 and 4, the implicit coupling is $O(\Delta t^2)$ accurate, it is unconditionally stable, and it leads to the exact steady state for constant forcing. Although from the stability and accuracy viewpoints the implicit coupling is very good, it nevertheless does have the important drawback of being difficult to implement in a computationally efficient manner in the context of a realistic atmospheric model. This motivates the symmetrized split-implicit coupling, which aims to achieve the advantages of the implicit coupling without compromising computational efficiency. It comprises the three-step discretization

$$\frac{F^*-F(t)}{\Delta t} = \frac{G-\beta F(t)}{2}, \tag{2.6}$$

$$\frac{F^{**}-F^*}{\Delta t}+\frac{i\alpha}{2}[F^{**}+F^*] = 0, \tag{2.7}$$

$$\frac{F(t+\Delta t)-F^{**}}{\Delta t} = \frac{G-\beta F(t+\Delta t)}{2}. \tag{2.8}$$

The first step is explicit and the other two are implicit. The third step, in the context of vertical diffusion, can again be accomplished by solving a set of tridiagonal problems in the vertical. The symmetrized split-implicit coupling comprises two physics discretizations arranged symmetrically around the dynamics discretization. This again keeps the physics discretization conveniently distinct from the dynamics one.

Eliminating F^* and F^{**} from (2.6)–(2.8) yields the equivalent coupling equation

$$\left(1+\frac{i\alpha\beta\Delta t^2}{4}\right)\left[\frac{F(t+\Delta t)-F(t)}{\Delta t}\right] + i\alpha\left[\frac{F(t+\Delta t)+F(t)}{2}\right] = G-\beta\left[\frac{F(t+\Delta t)+F(t)}{2}\right]. \tag{2.9}$$

This is very similar to what might be considered as the ideal implicit coupling equation (2.2) but with an $O(\Delta t^2)$ perturbation to the discrete time derivative. It is also reminiscent of, though quite distinct from, the two-time-level coupling scheme of the anelastic model of Grabowski and Smolarkiewicz (1996). This scheme achieves second-order accuracy by averaging the physics in a centered time fashion between times $t+\Delta t$ and t (in fact by averaging along a trajectory in their semi-

Lagrangian model) but they apply the averaged physics directly as a forcing to the dynamics in one step. As such they appear to be closer to the implicit scheme with its associated complexity and issues of efficiency. Another second-order-accurate scheme is the three-time-level hydrostatic model of Williamson and Olson (1994). Again the physics is applied directly as a forcing to the dynamics but now evaluated explicitly at the center time level. While attractive for some of the physics this scheme is unconditionally unstable for the diffusive process considered here. It seems likely, therefore, that Williamson and Olson (1994) in fact apply a somewhat different scheme for vertical diffusion though they do not discuss this aspect.

3. Stability and accuracy of the free solution

a. Stability

From (2.1), the free component of the explicit coupling satisfies

$$\frac{F^{\text{free}}(t+\Delta t) - F^{\text{free}}(t)}{\Delta t} + \frac{i\alpha}{2}[F^{\text{free}}(t+\Delta t) + F^{\text{free}}(t)] + \beta F^{\text{free}}(t) = 0. \tag{3.1}$$

Expanding $F^{\text{free}}(t)$ as

$$F^{\text{free}}(t) = F_0^{\text{free}} e^{i\omega t}, \tag{3.2}$$

then leads to

$$E = \frac{(1-\beta\Delta t) - i\alpha\Delta t/2}{1 + i\alpha\Delta t/2}, \tag{3.3}$$

where

$$E \equiv e^{i\omega\Delta t}. \tag{3.4}$$

Given that α ($\neq 0$) and β (≥ 0) are both real, and that $|E| \leq 1$ for stability, this yields the conditional stability condition

$$0 \leq \beta\Delta t \leq 2. \tag{3.5}$$

This condition is very restrictive for fast damping processes, such as vertical diffusion in the boundary layer at high resolution, thus rendering the explicit coupling computationally inefficient for practical applications. Indeed, although satisfaction of (3.5) guarantees stability of the free solution, the solution does not necessarily respect physical fidelity. This is particularly easy to see in the special case where $\alpha\Delta t$ is vanishingly small and $\beta\Delta t = 2$: (3.3) then reduces to $E = -1$ and $F^{\text{free}}(t)$ spuriously changes sign on alternate time steps. To not only avoid this problem but also guarantee that $|E|$ (the damping rate per time step) is a monotonically decreasing function of β (the damping coefficient), it is preferable to respect the more restrictive condition

$$0 \leq \beta\Delta t \leq 1. \tag{3.6}$$

The dispersion relations and associated stability conditions for the other three coupling schemes of section 2 may be found in a similar manner and the ensuing results are summarized in Table 1. While the explicit coupling scheme for the revised canonical problem has a very restrictive stability condition, the other three share the advantage of being unconditionally stable.

TABLE 1. Dispersion relation (E) and stability criterion as a function of coupling scheme.

Scheme	Dispersion relation (E)	Stability condition
Exact	$\exp[-(i\alpha+\beta)\Delta t]$	$0 \leq \beta\Delta t$
Explicit	$\dfrac{(1-\beta\Delta t) - i\alpha\Delta t/2}{1+i\alpha\Delta t/2}$	$0 \leq \beta\Delta t \leq 2$
Implicit	$\dfrac{(1-\beta\Delta t/2) - i\alpha\Delta t/2}{(1+\beta\Delta t/2) + i\alpha\Delta t/2}$	$0 \leq \beta\Delta t$
Split-implicit	$\left(\dfrac{1}{1+\beta\Delta t}\right)\left(\dfrac{1-i\alpha\Delta t/2}{1+i\alpha\Delta t/2}\right)$	$0 \leq \beta\Delta t$
Symmetrized split-implicit	$\left(\dfrac{1-\beta\Delta t/2}{1+\beta\Delta t/2}\right)\left(\dfrac{1-i\alpha\Delta t/2}{1+i\alpha\Delta t/2}\right)$	$0 \leq \beta\Delta t$

b. Accuracy of the free solutions

From (1.5), (3.2), and (3.4),

$$\begin{aligned} E_{\text{exact}} &\equiv \exp[i\omega_{\text{exact}}\Delta t] = \exp[-(i\alpha+\beta)\Delta t] \\ &= 1 - (i\alpha+\beta)\Delta t + \frac{(i\alpha+\beta)^2\Delta t^2}{2} \\ &\quad - \frac{(i\alpha+\beta)^3\Delta t^3}{6} + O(\Delta t^4), \end{aligned} \tag{3.7}$$

where $\omega_{\text{exact}} = -\alpha + i\beta$ is the exact value of ω.

For the explicit coupling, by expanding (3.3) in terms of Δt, the corresponding discrete expression is

$$\begin{aligned} E_{\text{explicit}} &= 1 - (i\alpha+\beta)\Delta t + \frac{i\alpha(i\alpha+\beta)\Delta t^2}{2} \\ &\quad + O(\Delta t^3) \\ &= E_{\text{exact}} - \frac{(i\alpha+\beta)\beta\Delta t^2}{2} + O(\Delta t^3). \end{aligned} \tag{3.8}$$

Thus the discrete transient amplitude only agrees with the exact one to $O(\Delta t)$ if $\beta \neq 0$, but to $O(\Delta t^2)$ if $\beta \equiv 0$, that is, when there is no damping term. The accuracy of the discrete transient amplitude ($E - E_{\text{exact}}$) for the other three coupling schemes may be found in a similar manner and the ensuing results are summarized in Table 2.

Examination of Table 2 shows that the split-implicit coupling has the same formal accuracy as the explicit coupling, while both the implicit and symmetrized split-implicit couplings are $O(\Delta t)$ more accurate for $\beta \neq 0$.

TABLE 2. $E - E_{exact}$ as a function of coupling scheme.

Scheme	$E - E_{exact}$
Explicit	$-(i\alpha + \beta)\beta\Delta t^2/2 + O(\Delta t)^3$
Implicit	$-(i\alpha + \beta)^3 \Delta t^3/12 + O(\Delta t)^4$
Split-implicit	$\beta^2\Delta t^2/2 + O(\Delta t)^3$
Symmetrized split-implicit	$-[(i\alpha)^3 + \beta^3]\Delta t^3/12 + O(\Delta t)^4$

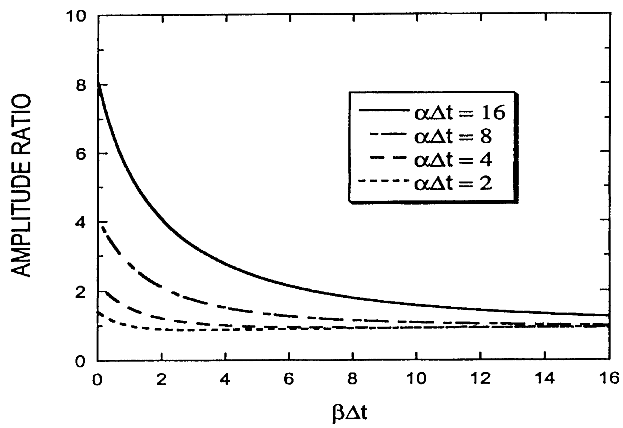

FIG. 1. The amplitude ratio, $|F^{forced}_{split-implicit}/F^{forced}_{exact}|$, as a function of the nondimensional parameter $|\beta\Delta t|$ for $\alpha\Delta t = 2, 4, 8$, and 16.

4. The forced response

The complete solution to each of (2.1), (2.2), (2.5), and (2.9) is the sum of the free response, F^{free}, given in section 3, and the forced response, F^{forced}, for each scheme. Since the forcing is constant a particular solution for each scheme is F^{forced} = constant. This will be equal to the steady-state solution, F^{steady}, when it exists.

a. Forced response of the explicit, implicit, and symmetrized split-implicit couplings

Seeking constant solutions, it is seen that the forced responses of (2.1), (2.2), and (2.9), respectively, for the explicit, implicit, and symmetrized split-implicit discretizations, are all identical to the exact one given by (1.6). This is not so, however, for the split-implicit coupling. This was noted by CLZ98 for the special case $\beta \equiv 0$.

b. Forced response of the split-implicit coupling

The forced response of (2.5) for the split-implicit coupling is

$$F^{forced}_{split\text{-}implicit} = \frac{G}{i\alpha/(1 + i\alpha\Delta t/2) + \beta}. \tag{4.1}$$

When $\beta \equiv 0$, and after rescaling Δt by a factor of 2 and making allowance for differences in notation, (4.1) is equivalent to the steady-state solution (15) of CLZ98. As noted by CLZ98 for the special case $\beta \equiv 0$, the discrete forced, or equivalently, steady-state response (4.1) now differs from the exact result (1.6). For large values of $|\alpha|$, that is, for rapidly propagating gravity and acoustic oscillations, this can be a significant source of error. As an example, CLZ98 consider a value of the buoyancy, or Brunt–Väisälä, frequency, $|\alpha|$, of 10^{-2} s^{-1} and, in their three-time-level discretization, choose a time step of 15 min. This corresponds to a time step of 30 min within the context of the present two-time-level discretization. This represents a relatively large time step, which is permitted by semi-implicit semi-Lagrangian models. With these choices $|\alpha\Delta t/2| = 9$ and, as CLZ98 estimated, the steady forced response is spuriously increased by an order of magnitude.

CLZ98 note the apparent paradox that their finding regarding spurious amplification of the forced response is at variance with results obtained in the context of two-time-level semi-implicit semi-Lagrangian models such as that of Chen and Bates (1996) and others. Since the level of vertical diffusion within such models is not generally insignificant, at least within the boundary layer, the role of β in (4.1) may be important. To examine this aspect, the ratio

$$\left|\frac{F^{forced}_{split\text{-}implicit}}{F^{forced}_{exact}}\right| = \left\{\frac{[1 + (\alpha\Delta t)^2/4][(\alpha\Delta t)^2 + (\beta\Delta t)^2]}{(\alpha\Delta t)^2(1 + \beta\Delta t/2)^2 + (\beta\Delta t)^2}\right\}^{1/2}, \tag{4.2}$$

of the amplitude of the approximate forced response (4.1) to the exact one (1.6), is plotted in Fig. 1 as a function of the nondimensional parameter $|\beta\Delta t|$ for $\alpha\Delta t = 2, 4, 8$, and 16. Figure 1 shows that the effect of strong time-implicit damping is to significantly reduce the spurious amplification of the forced response analyzed in CLZ98. This result could, at least partially, explain the paradox raised in CLZ98. Additionally, the present results suggest that, at the very least, the paradox is not due to issues associated with differences between two- and three-time-level schemes.

From (4.2) it can be seen that $|F^{steady}_{split-implicit}/F^{steady}_{exact}|$ is less than or greater than unity according to whether $\beta\Delta t$ is greater or less than the critical value $(\alpha\Delta t/2)^2$. Thus a sufficiently large (but possibly physically unrealizable) damping can not only inhibit the spurious amplification of the constant forced component of the flow, but can also spuriously diminish it. Let F oscillate in the vertical with a vertical wavenumber m. Then, for vertical diffusion, $\beta = m^2\nu$, where ν is the appropriate diffusion coefficient for F. The vertical wavenumber is bounded above by $\pi/\Delta z$, where Δz is the local vertical mesh length, and so a model with high vertical resolution in the planetary boundary layer can lead to very large values of $|\beta\Delta t|$ for highly turbulent flow. Assuming a time step of 30 min and taking $|\alpha| = 10^{-2}$ s^{-1}, as above, gives a critical value of $\beta \simeq 5 \times 10^{-2}$ s^{-1} above which the forced response will be less than the exact one. A value of $\nu = 5$ m^2 s^{-1}, as in Teixeira (2000), is reasonable for neutral and weakly stable boundary layers. Then, using the largest permitted wavenumber, $\pi/\Delta z$, it is found that values of β larger than the critical value are likely to occur for mesh lengths smaller than about 30 m. Note that for very large values of $\beta\Delta t$ the amplitude of the approximate forced response (4.1) asymptotes to the exact one (1.6).

As noted by a reviewer, more centered versions of (2.4) could be obtained by replacing $\beta F(t + \Delta t)$ by either (a) $\beta[F(t + \Delta t) + F(t)]/2$, or by (b) $\beta[F(t + \Delta t) + F^*]/2$. However both possibilities still spuriously corrupt the forced response. Possibility (a) again leads to (4.1) and (4.2). For possibility (b), the ratio of the amplitude of the approximate forced response to the exact one is $[1 + (\alpha\Delta t/2)^2]^{1/2}$. This is independent of β and in fact identical to the expression given in (4.2) but with β set identically to zero. This further worsens the spurious forced response: the ratio of the amplitude of the approximate forced response to the exact one no longer responds to any increase in the nondimensional damping coefficient $\beta\Delta t$, and it no longer asymptotes to the exact value of unity at large $\beta\Delta t$.

5. Summary and conclusions

The canonical problem that CLZ98 used to study numerical aspects of coupling physics to dynamics has been modified to consider simultaneously the impact of an oscillatory term representative of the dynamics and a damping term representative of a physics diffusion scheme, as well as a constant forcing term. The full transient solutions to the resulting discrete equations were considered for four coupling strategies. Here a two-time-level discretization was employed in contrast to CLZ98 who used a three-time-level discretization. For the two coupling schemes considered by CLZ98 (the explicit and split-implicit), it was shown that the different number of time levels does not impact on the conclusions they obtained. Aspects of each of the coupling strategies are summarized below.

- The ‘‘explicit’’ coupling has the advantages of simplicity and correct representation of the exact forced solution, but suffers from the important disadvantage that the time step is limited by both the stability condition (3.5), which is very restrictive for fast processes such as vertical diffusion of the boundary layer at high resolution, and the $O(\Delta t)$ discretization of the $-\beta F$ damping term.
- The ‘‘implicit’’ coupling fully addresses the stability and accuracy deficiencies of the explicit coupling and, again, correctly represents the exact steady-state solution. It does however have the drawback that it is very difficult in the full-model context to efficiently solve the coupled set of equations arising from the semi-implicit discretization. This is due to the introduction of additional terms that inhibit the usual elimination procedure to obtain an elliptic-boundary-value problem for a single variable.
- The ‘‘split-implicit’’ coupling addresses the stability issue of the explicit coupling but, as argued in CLZ98, it does so at the expense of accuracy. One $O(\Delta t)$ discretization has effectively been replaced by another, and the resulting truncation error is still large for large Δt. In particular the split-implicit coupling corrupts the forced response, which can, as identified in CLZ98 in the context of the forced steady-state response, be spuriously amplified by an order of magnitude. However a strong time-implicitly treated damping mechanism, such as vertical diffusion, can significantly decrease the seriousness of this latter problem to the point of even underestimating the forced response. This could help explain the paradox raised in CLZ98’s conclusions that their finding regarding spurious amplification of the forced response is at variance with results obtained by others.
- The novel ‘‘symmetrized split-implicit’’ coupling can fully address the stability and accuracy deficiencies of the explicit coupling while keeping the physics discretization distinct from the dynamics one and can also correctly represent the steady-state solution. This then gives neutral stability of the free solution in the absence of damping, and the discretization is $O(\Delta t^2)$ accurate. The only apparent drawback, suffered also by the implicit and split-implicit couplings, and an arguably serious one, is the difficulty in the full-model context of efficiently achieving an implicit discretization of all the physics terms in the final physics substep. This difficulty may be circumvented, however, if the physics columns are mutually independent, since in the symmetrized split-implicit scheme (and also in the less accurate split-implicit one) the implicitness of the problem would then be reduced from three dimensions to one. Then an iterative approach, with a small number of iterations is perhaps viable, provided this is cheaper than simply reducing the time step for the entire model. This would have the advantage of maintaining consistency and net balance between different physical processes.

It is important of course to bear in mind that the problem considered here, based on the canonical problem of CLZ98, is a considerable simplification of any real physics–dynamics coupling. Such simplification allows detailed analysis of the scheme but overlooks the complications associated with the full problem. Such complications include (a) the highly coupled (in the sense now of coupled prognostic equations) nature of the equation set that makes implicit schemes, if not intractable, certainly much harder and potentially more expensive than might appear to be the case from the schemes considered here; (b) nonlinearity of the full models [it is worth noting though that the additional problems associated with the nonlinearity introduced when the diffusivity, related here to β, depends on the prognostic variables, have been successfully considered in a simplified context by, for example, Kalnay and

Kanamitsu (1988) and Bénard et al. (2000)]; and (c) the complexity of a package of physical parameterizations with a spectrum of timescales that leads to the notion of process splitting versus time splitting of the various parameterizations and associated issues (see Williamson 1999). Nevertheless, the continued improvement of weather and climate prediction models relies on an ability to exploit improvements in physical understanding and associated developments in the parameterizations to their fullest. This itself depends critically on understanding the issues associated with the robustness and accuracy of the coupling between the dynamics and the physics and more focused study of the forced response of the systems of equations. The simplified, canonical problems such as those introduced by CLZ98 and developed further here seem an essential starting point for the development of such understanding. A follow-up study, to be published elsewhere, has been conducted in which the model problem used herein is further generalized to include a spatiotemporal forcing, and further insight into physics–dynamics coupling issues is obtained by examining its free and forced responses.

Acknowledgments. Helpful reviews of this note by two anonymous reviewers, and by Mike Bell of a draft version, are gratefully acknowledged.

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