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A Mechanistic Model of Eulerian, Langrangian Mean, and Lagrangian Ozone Transport by Steady Planetary Waves

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Ozone transport is calculated for steady, dissipative planetary waves using the Eulerian, Lagrangian mean, and residual circulation. A Langrangian model of parcel dynamics is used to interpret planetary wave-photochemistry interaction. In chemically active regions the mean field ozone changes are found to be significant only where there are large gradients in chemical sources and sinks along parcel trajectories. The largest changes in the mean field are found in the lower stratosphere and are due to the Lagrangian mean advection. When the Lagrangian mean advection is approximated by the residual circulation, errors in the transport velocities as large as 30% may occur.

1. INTRODUCTION

The atmospheric ozone distribution is maintained by a complex interaction of photochemical, dynamical, and radiational processes. In order to make dynamical modeling of ozone tractable, it has been the practice to parameterize severely either the photochemistry or the dynamics. For instance, one-dimensional (1-D) chemistry models have used vertical diffusion to model atmospheric dynamics [*Hunten*, 1975 and the references therein], while two-dimensional (2-D) models have involved a combination of a mean advective field with parameterized eddy dynamics [*Harwood and Pyle*, 1977; *Miller et al.*, 1981]. The wave transport by planetary waves in 2-D models has usually been parameterized as a symmetric diffusion tensor. However, planetary wave effects have been shown to be largely advective [*Matsuno*, 1980; *Dunkerton*, 1980; *Pyle and Rogers*, 1980; *Danielsen*, 1981; *Miller et al.*, 1981; *Strobel*, 1981]; therefore the parameterized dynamics have often been very unrealistic [*NASA*, 1979]. Since the transport of ozone away from source regions is important in the maintenance of the ozone distribution, it is necessary to use realistic dynamic processes in ozone models.

In order to identify the mechanisms responsible for ozone transport and therefore the proper dynamical processes to be included in chemical models, a series of studies involving simple dynamics and photochemistry has been undertaken. In this paper the effects of steady, dissipative planetary waves on the ozone distribution will be discussed. The problem will be formulated in both the Eulerian and Lagrangian mean systems and the relation of the Lagrangian mean to the residual circulation will be investigated. In a subsequent paper the effects of time-dependent planetary wave/mean flow fields on the ozone distribution will be examined.

In section 2 the dynamical model is described. In secton 3 the continuity equation is developed for small-amplitude waves in both the Eulerian and Lagrangian mean formulations. Also the relation between the residual circulation and the Lagrangian mean velocity is derived. In section 4, model results are presented, and a Lagrangian model based on particle trajectories is given.

2. DYNAMIC MODEL

The planetary waves are modeled by solving the steady, linear quasi-geostrophic potential vorticity equation on a mid-latitude β plane. The governing equation is

$$\left(a + \bar{u}\frac{\partial}{\partial x}\right)\left\{\nabla^2\phi' + \frac{f^2}{\rho N^2}\frac{\partial}{\partial z}\rho\phi_z'\right\} + \bar{q}_y\frac{\partial\phi'}{\partial x} = 0 \qquad (1)$$

It is assumed that the mean flow is steady and governed by

$$a\left\{\frac{\partial^2\bar{\phi}}{\partial y^2} + \frac{f^2}{\rho N^2}\frac{\partial}{\partial z}\rho\bar{\phi}_z\right\} = f\frac{\partial^2}{\partial y^2}\overline{u'v'} - \frac{f^2}{\rho N^2}\frac{\partial}{\partial z}\rho\frac{\partial}{\partial y}\overline{v'\phi_z'} \qquad (2)$$

The mean meridional circulation ($\bar{v}$ and $\bar{w}$) is forced by the wave field (see notation section). It is assumed that wave effects on the mean flow are not so large that the wave field needs to be recomputed with the perturbed mean flow. This restricts the validity of the model to small-amplitude waves and is consistent with the transport approximations to be made later. The model is essentially the Eulerian analogue to the Lagrangian mean model of *Schoeberl* [1981].

Damping (a in (1) and (2)) is modeled by Newtonian cooling and Rayleigh friction, which are assumed to be equal. The damping is a slowly varying function of height except in the upper most regions where it is assumed to increase at an exponential rate. This region of increase provides a sponge layer to reduce reflections from the upper boundary. The time scale of the damping below the sponge layer is approximately 10 days, a number representative of the middle or upper stratosphere [*Schoeberl and Strobel*, 1978; *Dickinson*, 1973].

3. CONSTITUENT DYNAMICS

The chemistry used in this study is extremely simple, namely,

$$Q = -\lambda(\mu - \overline{\mu_0}) \qquad \lambda > 0 \qquad (3)$$

where $\overline{\mu_0}$ is the zonal mean basic state. This formulation requires that the perturbations relax back to zero, while changes

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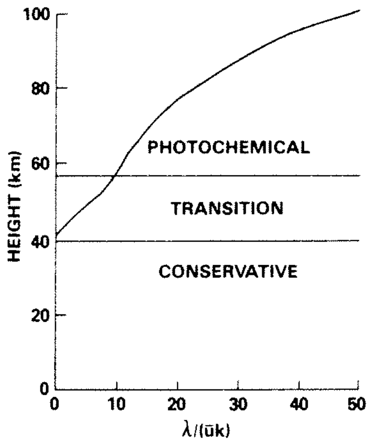

Fig. 1. Ratio of dynamical to photochemical time scales, $\lambda/\bar{u}k$, showing the atmosphere divided into conservative, transition, and photochemical regions.

in the mean field relax to the mean equilibrium. The rate of relaxation is inversely proportional to λ, which is assumed to be a function of height only. The λ values are based on those of *Hartmann and Garcia* [1979].

The chemical reactions that the λ values are based on are the oxygen only equations with a modified ozone destruction rate constant to represent catalytic destruction. The chemistry is temperature dependent; however, the temperature dependence will be neglected to aid interpretation. *Hartmann and Garcia* [1979] and *Garcia and Hartmann* [1980] found that the temperature effects were significant and enhanced the eddy transport The temperature dependence is less important in this model due largely to smaller calculated wave amplitudes.

In order to discuss the transport of ozone by stationary waves the atmosphere is divided into three regions, as shown in Figure 1. The lower region is the region in which ozone is assumed to be a conservative tracer. In the upper region, ozone photochemistry is so fast (λ large) that the transport is unimportant. The middle region is defined by the ration $0.1 < \lambda/\bar{u}k < 10$ and is called the transition region [*Hartmann and Garcia*, 1979].

Figure 2 shows the ozone mixing ratio used in this model, which is based on the ozone distribution used by *Hartmann and Garcia* [1979]. The use of the Hartmann and Garcia ozone distribution and chemistry facilitates comparison with their results.

The ozone continuity equation is formulated in both the Eulerian and Lagrangian mean systems. In general, the constituent continuity equation is

$$D\mu/Dt = Q \tag{4}$$

where Q represents the source term. By decomposing the fields into zonal averages and deviations and linearizing, the Eulerian formulation is obtained:

$$\frac{\partial\bar{\mu}}{\partial t} + \bar{u}_i\frac{\partial\bar{\mu}}{\partial x_i} + \overline{u_i'\frac{\partial\mu'}{\partial x_i}} = \bar{Q} \tag{5}$$

$$\frac{\partial\mu'}{\partial t} + u_i'\frac{\partial\bar{\mu}}{\partial x_i} + \bar{u}_i\frac{\partial\mu'}{\partial x_i} = Q' \tag{6}$$

where repeated indices indicate summation. It is pertinent to note that the zonal mean equation contains eddy flux terms, which represent the effects of wave transport. The tendency for the eddy transport to be counteracted by the advection by the mean flow has been discussed extensively and represents a major difficulty of atmospheric transport problems [*Dunkerton*, 1980; *Mahlman and Moxim*, 1978].

Using the Lagrangian mean formulation [*Andrews and McIntyre*, 1978], it is possible to redefine the averaging operator such that wave forcing does not appear in the mean equation. Using this concept, the mean continuity equation is written as

$$\left(\frac{\partial}{\partial t} + u_i^L\frac{\partial}{\partial x_i}\right)\mu^L = Q^L \tag{7}$$

In this system the time rate of change of the Lagrangian mean ozone distribution is equal to the advection by the Lagrangian mean velocity plus a source term. Two features of the Lagrangian mean velocity as defined by Andrews and McIntyre are (1) for the premises of the noninteraction theorem [*Boyd*, 1976; *Dunkerton*, 1980] the Lagrangian mean velocity is zero, and (2) the Lagrangian mean velocity is the center of mass velocity of a particular ensemble of fluid particles.

With the second feature above, it may seem natural to study tracer transport in the Lagrangian mean system. However, it turns out the Lagrangian mean transform is complicated and is easily formulated only for the most simple flows [*McIntyre*, 1980]. Furthermore, dispersion of the particles about their center of mass in nonlinear and time-dependent problems may render the location of the center of mass of little use in the actual location of the tracer [*Hsu*, 1980].

The difference between the Lagrangian mean and the Eulerian zonal mean, for an arbitrary quantity γ, is defined as the Stokes correction:

$$\gamma^s \equiv \gamma^L - \bar{\gamma}$$

Relating the Lagrangian mean transform to the Eulerian zonal mean by the premise that the unperturbed fluid would be described by the Eulerian flow, it is possible to define the Stokes correction to second order as

$$\gamma^s = \overline{\xi_i\frac{\partial\gamma'}{\partial x_i}} + \tfrac{1}{2}\overline{\xi_i\xi_j}\frac{\partial^2\bar{\gamma}}{\partial x_i\,\partial x_j}$$

where $\xi_i = (\xi_1, \xi_2, \xi_3) = (\xi, \eta, \zeta)$ are the displacements from the equilibrium position of the fluid.

The displacement fields are related to the Eulerian perturbation velocity fields by the following equation [*Matsuno and Nakamura*, 1979]:

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\xi_i = u_i' + \xi_j\frac{\partial}{\partial x_j}\bar{u}_i + \cdots \tag{8}$$

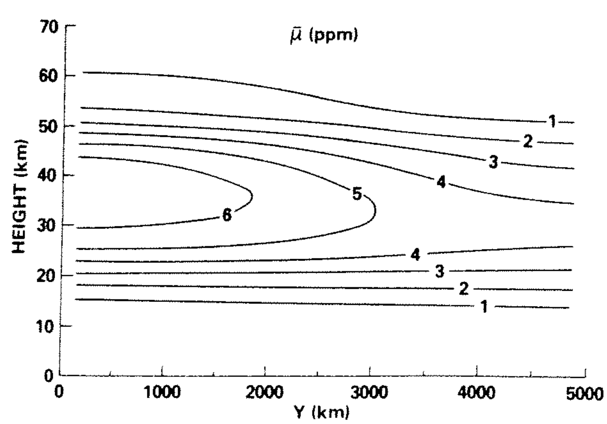

Fig. 2. Zonal mean ozone distribution (ppm).

The mean meridional circulation is forced by the waves and is assumed to be second order in wave amplitude ($O(a^2)$). Therefore the changes in the mean ozone field are $O(a^2)$, and the terms involving $\bar{v}$, $\bar{w}$ in (6) and (8) can be ignored. The perturbation continuity equation can now be written to $O(a^2)$ as

$$\frac{\partial \mu'}{\partial t} + \bar{u}\frac{\partial \mu'}{\partial x} + v'\frac{\partial \bar{\mu}_0}{\partial y} + w'\frac{\partial \bar{\mu}_0}{\partial z} = -\lambda\mu' \tag{9}$$

Using the mass continuity equation, (5) may be written as

$$\frac{\partial \bar{\mu}}{\partial t} + \frac{\partial}{\partial y}(\bar{v}\bar{\mu}_0 + \overline{v'\mu'}) + \frac{1}{\rho}\frac{\partial}{\partial z}\rho(\bar{w}\bar{\mu}_0 + \overline{w'\mu'}) = -\lambda\bar{\mu} \tag{10}$$

where $\bar{\mu}$ represents changes in the mean field from the equilibrium value $\bar{\mu}_0$.

The relationship between the perturbation velocity fields and the displacement fields can be rewritten as

$$ik\bar{u}\xi = u' + \eta'\frac{\partial \bar{u}}{\partial y} + \zeta'\frac{\partial \bar{u}}{\partial z}$$

$$ik\bar{u}\eta = v' \qquad ik\bar{u}\zeta = w'$$

These are the equations used to calculate the Lagrangian mean displacements.

Under the assumptions above, the following relationships between the Eulerian and the Lagrangian mean quantities can be written for steady waves:

$$\frac{\partial \mu^L}{\partial t} = \frac{\partial \bar{\mu}}{\partial t} + O(a^4) \tag{11a}$$

$$v^s \equiv \overline{\xi_i\frac{\partial v'}{\partial x_i}} + O(a^4) \tag{11b}$$

$$w^s = \overline{\xi_i\frac{\partial w'}{\partial x_i}} + O(a^4) \tag{11c}$$

$$\bar{Q}^L = \bar{Q} + \overline{\xi_i\frac{\partial Q'}{\partial x_i}} \tag{11d}$$

Therefore (7) can be written to the same accuracy as (10) as

$$\frac{\partial \bar{\mu}}{\partial t} = -u_i^L\frac{\partial \bar{\mu}_0}{\partial x_i} - \overline{\xi_i\frac{\partial \lambda\mu'}{\partial x_i}} - \lambda\bar{\mu} \tag{12}$$

The first term on the right-hand side of (12) will be referred to as the advective term; the second term is the 'stirring' term; and the third term is the same as the Eulerian mean chemistry. The stirring term represents the effects of the planetary waves moving ozone parcels through a region of varying photochemistry. This term will be discussed in detail later. Equations (12), (10), and (9) are the ozone continuity equations used in this study.

A number of authors have suggested that the Lagrangian mean velocity may be a suitable quantity to approximate the large-scale transport of ozone in 2-D chemical models [*Dunkerton*, 1978; *Miller et al.*, 1981; *McIntyre*, 1980]. However, since the Lagrangian mean velocity can be difficult to calculate, it has also been suggested that the residual circulation defined by *Andrews and McIntyre* [1976] might be an adequate approximation for the Lagrangian mean velocity [*Holton*, 1981]. *Strobel* [1981] has derived a general linearized 2-D transport model by using the residual circulation for waves with complex frequencies. The relation between the residual velocity and the Lagrangian mean velocity for steady, dissipative planetary waves is outlined below.

The eddy mass continuity equation can be written in terms of the parcel displacement fields as

$$\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{1}{\rho}\frac{\partial \rho\zeta}{\partial z} = 0 \tag{13}$$

Using (13) and the definition of the Stokes corrections, it is possible to write

$$v^L = \bar{v} + \frac{1}{\rho}\frac{\partial}{\partial z}\overline{\rho v'\zeta} \tag{14}$$

$$w^L = \bar{w} - \frac{\partial}{\partial y}\overline{v'\zeta} \tag{15}$$

The residual circulation for the current problem is

$$v^* = \bar{v} - \frac{1}{\rho N^2}\frac{\partial}{\partial z}\rho\,\overline{v'\phi_z'} \tag{16}$$

$$w^* = \bar{w} + \frac{1}{N^2}\frac{\partial}{\partial y}\overline{v'\phi_z'} \tag{17}$$

Using the eddy thermodynamic equation

$$(a + \bar{u}ik)\phi_z' - fv'\frac{\partial \bar{u}}{\partial z} + \bar{u}ikN^2\zeta = 0$$

and multiplying by v' and averaging, it can be shown that

$$\overline{v'\phi_z'} = \frac{-(k\bar{u})^2N^2\,\overline{v'\zeta} + a[f(\partial\bar{u}/\partial z)\,\overline{v'v'} - N^2\,\overline{v'w'}]}{a^2 + (k\bar{u})^2} \tag{18}$$

where it has been assumed that only the correlations of real parts are considered. Using (18) and comparing (14) and (15) with (16) and (17), it is clear that the Lagrangian mean velocity and the residual velocity are equal only when $a = 0$. When $a = 0$, however, both the Lagrangian mean and the residual circulation are zero, and the transport by the waves and the mean flow exactly counterbalance each other. Since a is an order of magnitude or more smaller than $k\bar{u}$ below the sponge layer in this model, the Lagrangian mean and residual circulations should be similar. When transience or external heating and cooling processes are allowed the relationship between the two circulations becomes more complex. In *Holton*'s [1981] parameterization the difference between the Lagrangian mean and residual circulations is contained in the antisymmetric part of the diffusion tensor and is assumed to be negligible.

In the next section the Eulerian velocities, the Lagrangian mean quantities, and the residual circulation will be presented for steady, dissipating planetary waves. Zonal mean ozone changes can then be calculated, and the various transport methods can be compared.

4. RESULTS

4.1. *Dynamic Quantities*

Equations (1) and (2) are solved for a wave number one perturbation with forcing analogous to a mean zonal wind of 10 m/s flowing over a 1-km mountain. The mean zonal wind profile and the geopotential amplitude are shown in Figure 3. The mean zonal wind is presumed to approximate the polar night jet. The geopotential reaches a maximum amplitude of about 1 km in the mesosphere and does not exceed the steady state limit [*Schoeberl*, 1982]. The magnitude of the geopotential is in rough agreement with observation [*van Loon et al.*, 1973]. The eddy geopotential field is used to calculate the wave forced mean meridional and vertical velocity.

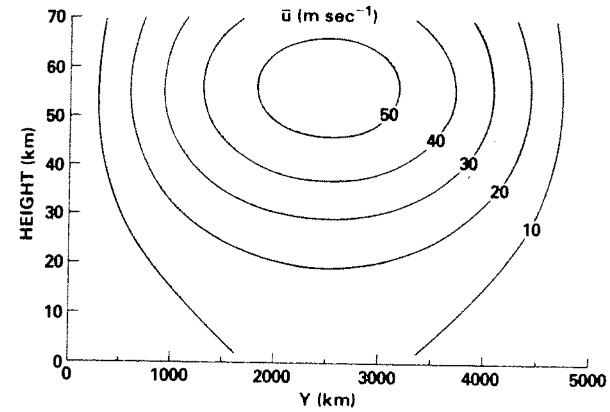

Fig. 3*a*

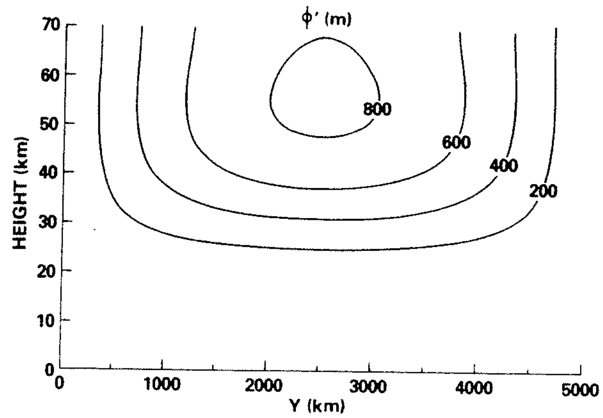

Fig. 3*b*

Fig. 3. (*a*) Mean zonal wind $\bar{u}$ (m/s). (*b*) Amplitude of wave number one geopotential (m).

The $\bar{v}$ and $\bar{w}$ fields are shown in Figure 4. These fields, as expected, show rising motion in the northern regions of the β channel and sinking motion in the southern regions. The meridional velocity is from north to south. This flow is forced by the northward eddy heat transport of the planetary waves.

The Stokes drifts as calculated with (11*b*) and (11*c*) are shown in Figure 5. These drifts can be viewed as the effective velocity fields associated with the wave. The Stokes drifts are directed in an opposite sense to the Eulerian mean velocity and are of approximately the same magnitude. This demonstrates the tendency for the wave fields to counterbalance the Eulerian mean fields, as has been discussed by *Matsuno* [1980].

The Lagrangian mean velocity fields, which are the sums of the Eulerian zonal mean and the Stokes drifts, are shown in Figure 6. The sense of the circulation is rising motion in the southern regions and sinking motion in the polar regions. This motion is the same as indicated by *Schoeberl* [1981]. Furthermore, this motion would tend to enhance the diabatic circulation as calculated by *Dunkerton* [1978]. The Lagrangian mean velocity is of much smaller magnitude than either the Stokes drifts or the Eulerian mean velocities.

The residual circulation is shown in Figure 7. It is geometrically similar to the Lagrangian mean velocity, but the meridional component is approximately 30% larger than the Lagrangian mean meridional velocity. The largest differences between the two circulations occur in the region where $\partial\bar{u}/\partial z$ is largest (see equation (18)). Similar increases are calculated in the vertical residual velocity field.

4.2. *Eulerian Transport*

The continuity equations (9), (10), and (12) are solved using the appropriate dynamic quantities. Since the mean chemistry appears in the same form in both (10) and (12), it is ignored in order that it might not obscure the differences between the two formulations. The effects of the mean chemistry term will be considered separately.

The horizontal ($\bar{v}\bar{\mu} + \overline{v'\mu'}$) and vertical ($\bar{w}\bar{\mu} + \overline{w'\mu'}$) fluxes as calculated by the Eulerian method are shown in Figure 8. The vertical flux is upward in the northern region of the model and downward in the southern region. The vertical flux is confined primarily to the conservative region and reflects the Eulerian mean vertical velocity field.

The horizontal flux field is more complicated, showing two regions of northward transport separated by a region of southward transport. Once again the flux is largest in the conservative region. The lower region of northward flux is dominated by the eddy flux, which is proportional to the vertical derivative of the mean constituent distribution, as described by *Clark and Rogers* [1978]. The region of southward flux, which occurs where the vertical gradient of the constituent density is small, is dominated by the mean meridional velocity. The upper region of northward transport is once again dominated by the eddy flux. In the upper most regions of the model, both the horizon-

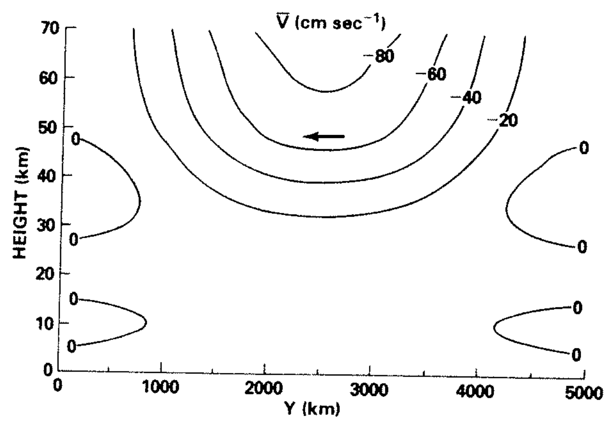

Fig. 4*a*

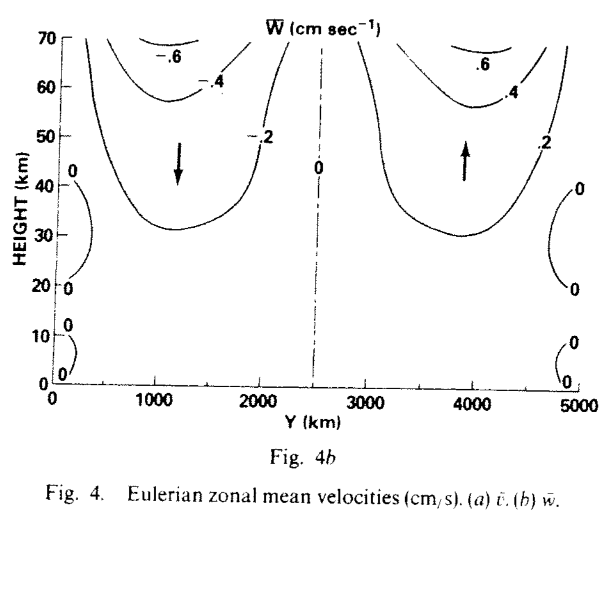

Fig. 4*b*

Fig. 4. Eulerian zonal mean velocities (cm/s). (*a*) $\bar{v}$. (*b*) $\bar{w}$.

tal and vertical fluxes are very small due to rapid photochemical relaxation.

The rate of change of the mean ozone field is calcuated as the divergence of the fluxes. This field is shown in Figure 9. There is an increase of ozone in the northern section and a decrease in the southern section. Despite the fluxes being largest in the conservative region the mean field tendency is concentrated almost entirely in the transition region in agreement with the results of *Hartmann and Garcia* [1979].

4.3. *Lagrangian Mean Transport*

The Lagrangian mean results are shown in Figures 10–12. The advective term as defined in (12) is shown in Figure 10. In the extreme polar region there is a decrease of ozone above the ozone maximum and a buildup of ozone in the lower stratosphere. In the southern regions there is a decrease in the lower stratosphere and an increase in the upper stratosphere. These changes are primarily due to the vertical advection. There is a strong transport of ozone northward by the meridional Lagrangian mean velocity in the mid-latitudes. The Lagrangian mean velocity associated with planetary waves advects ozone poleward and downward.

The advective term calculated using the residual circulation shows increases and decreases in the mean ozone field in the same regions as the Lagrangian mean calculations. In all regions the magnitude of advection by the residual circulation is larger. Differences as large as 30% are calculated reflecting the increased magnitude of the residual circulation.

Comparison of Figures 11 and 9 show that the mean field changes are the same when calculated by the Eulerian or Lagrangian mean methods. Since the advective term is much smaller than the stirring term, substituting the residual circulation for the Lagrangian mean velocity shows only minute changes in the tendency field.

4.4. *The 'Stirring' Term*

The magnitude of the stirring term is shown in Figure 12. This field quite accurately represents the ozone time rate of change shown in Figure 11. The advection by the Lagrangian mean velocity is less than 1% of the stirring term in regions where ozone is photochemically active. The stirring term is identically zero when the photochemistry is zero.

In order to understand the physical meaning of the stirring term and its analogy to the Eulerian problem, it is helpful to compare the stirring term to the eddy flux. Assuming that the wave is steady and stationary, (9) becomes

$$\mu' = -\frac{\lambda - ik\bar{u}}{\lambda^2 + (k\bar{u})^2} u_i' \frac{\partial \bar{\mu}_0}{\partial x_i} \tag{19}$$

The change in the zonal mean ozone field due to the divergence

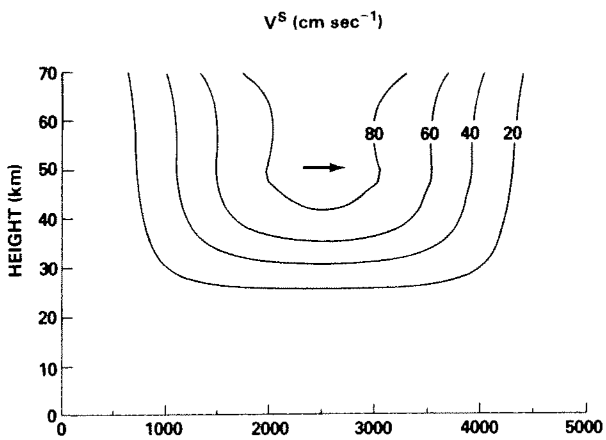

Fig. 5*a*

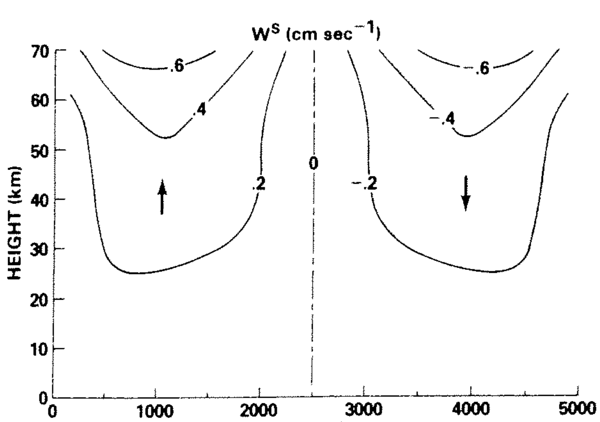

Fig. 5*b*

Fig. 5. Stokes drifts (cm/s). (*a*) v^s. (*b*) w^s.

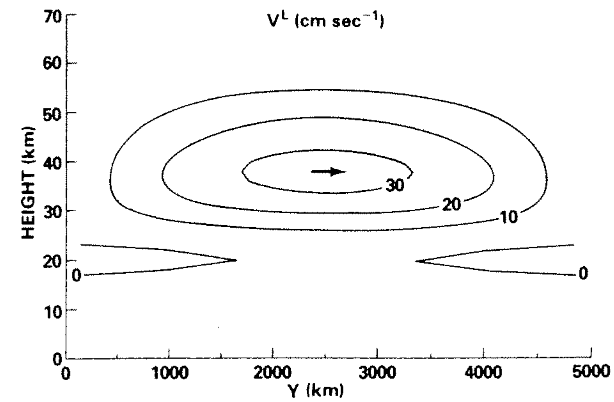

Fig. 6*a*

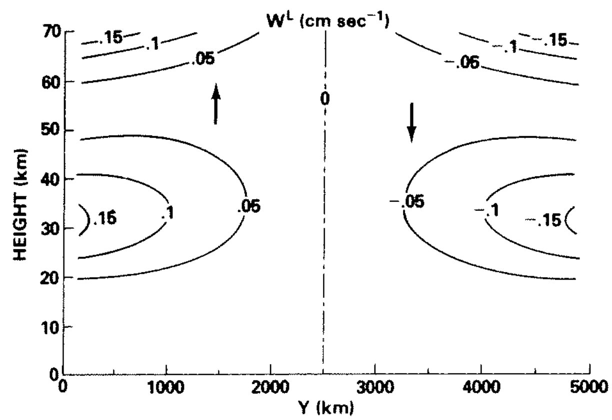

Fig. 6*b*

Fig. 6. Lagrangian mean velocities (cm/s). (*a*) v^L. (*b*) w^L.

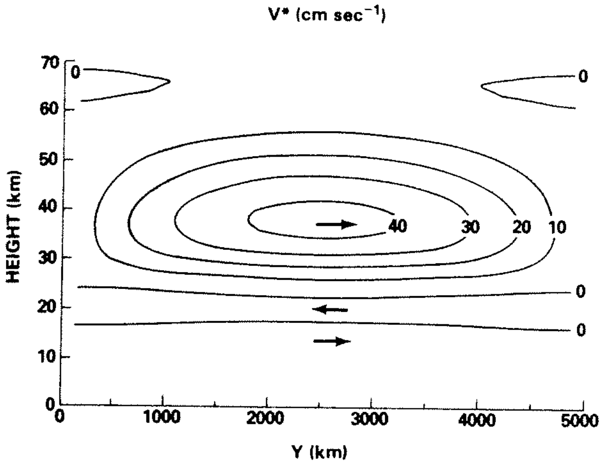

Fig. 7a

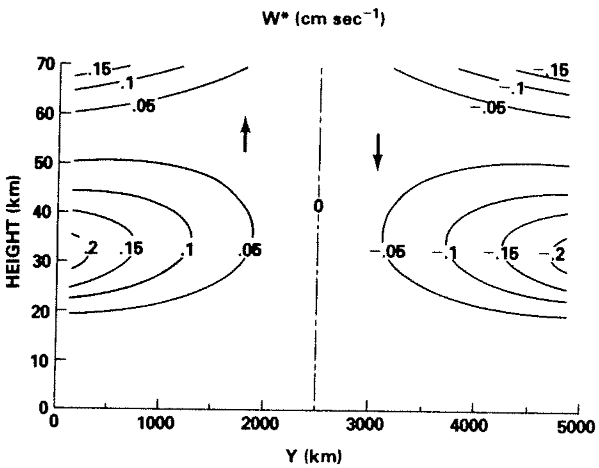

Fig. 7b

Fig. 7. Residual velocities (cm/s). (a) v^*. (b) w^*.

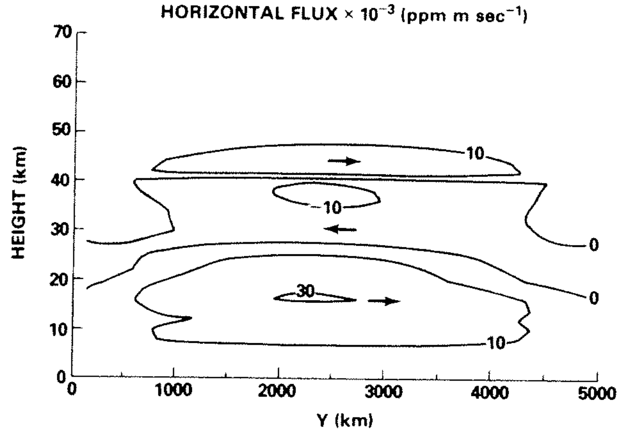

Fig. 8a

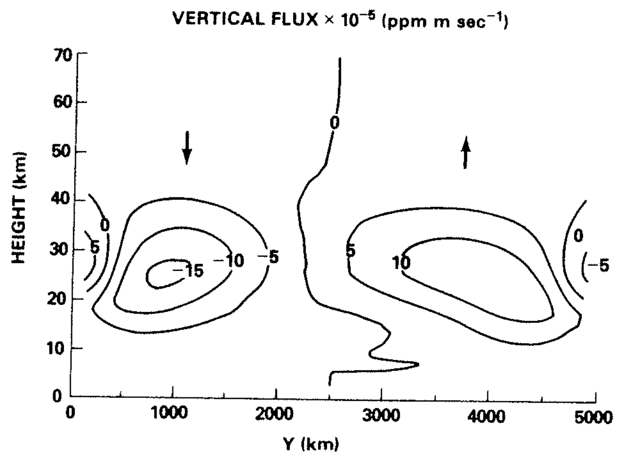

Fig. 8b

Fig. 8. Eulerian fluxes of ozone (ppm m/s). (a) Horizontal flux. (b) Vertical flux.

of the eddy flux (DEF) is

$$\mathrm{DEF} = -\frac{1}{\rho}\frac{\partial}{\partial x_j}\rho\,\overline{u_j'\mu'}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial x_j}\rho\left\{\frac{\lambda}{\lambda^2+(k\bar{u})^2}\,\overline{\mathrm{Re}(u_j')\mathrm{Re}(u_i')}\,\frac{\partial\bar{\mu}_0}{\partial x_i} + \frac{k\bar{u}}{\lambda^2+(k\bar{u})^2}\,\overline{\mathrm{Re}(u_j')\mathrm{Im}(u_i')}\,\frac{\partial\bar{\mu}_0}{\partial x_i}\right\} \qquad (20)$$

The stirring term (ST) is

$$\mathrm{ST} = -\overline{\xi_j\frac{\partial\lambda\mu'}{\partial x_j}} = -\frac{1}{\rho}\frac{\partial}{\partial x_j}\overline{\rho\lambda\xi_j\mu'}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial x_j}\frac{\rho\lambda}{k\bar{u}}\left\{\frac{\lambda}{\lambda^2+(k\bar{u})^2}\,\overline{\mathrm{Im}(u_j')\mathrm{Re}(u_i')}\,\frac{\partial\bar{\mu}_0}{\partial x_i} + \frac{k\bar{u}}{\lambda^2+(k\bar{u})^2}\,\overline{\mathrm{Re}(u_j')\mathrm{Re}(u_i')}\,\frac{\partial\bar{\mu}_0}{\partial x_i}\right\} \qquad (21)$$

ST is functionally very similar to DEF. In order to investigate the precise similarities it is useful to consider the limits of λ relative to the dynamic frequency $k\bar{u}$.

For $\lambda/k\bar{u} < 1$,

$$\frac{1}{\lambda^2+(k\bar{u})^2} \cong \frac{1}{(k\bar{u})^2}\left(1-\frac{\lambda^2}{(k\bar{u})^2}\right) \qquad (22)$$

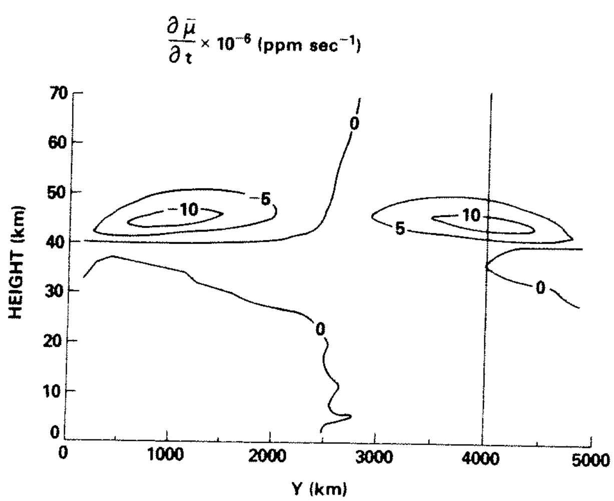

Fig. 9. Mean ozone tendency $\partial\bar{\mu}/\partial t$, as calculated with the Eulerian formulation (ppm/s).

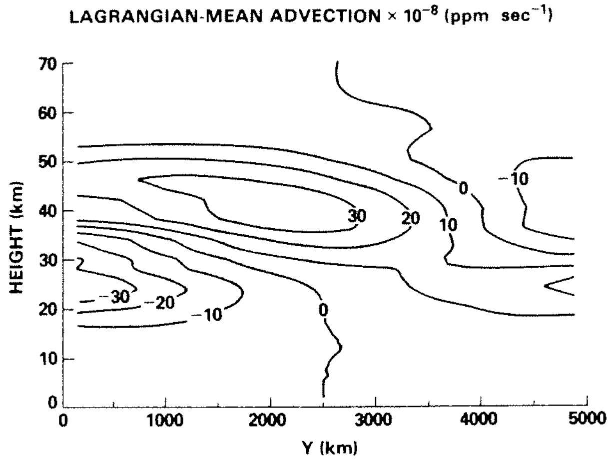

Fig. 10*a*

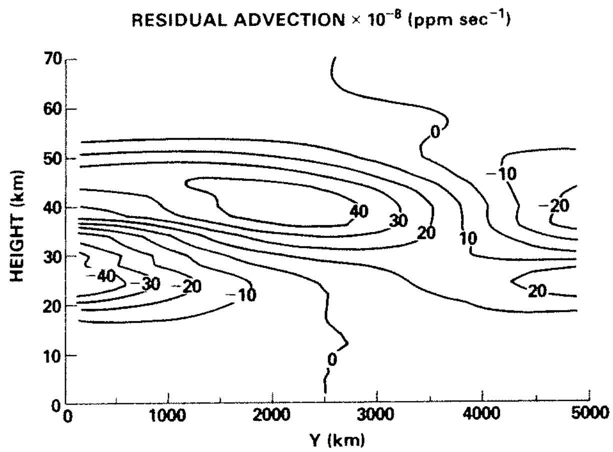

Fig. 10*b*

Fig. 10. Mean ozone tendency caused by advection (ppm/s). (*a*) Lagrangian mean velocity. (*b*) Residual velocity.

Using (22), (20) and (21) can be rewritten to lowest order as

$$\mathrm{DEF} = -\frac{1}{\rho}\frac{\partial}{\partial x_j}\,\rho\left\{\frac{\lambda}{(k\bar{u})^2}\,\overline{\mathrm{Re}(u_j')\mathrm{Re}(u_i')}\,\frac{\partial \bar{\mu}_0}{\partial x_i} + \frac{1}{(k\bar{u})}\,\overline{\mathrm{Re}(u_j')\mathrm{Im}(u_i')}\,\frac{\partial \bar{\mu}_0}{\partial x_i}\right\} \tag{23}$$

and

$$\mathrm{ST} = -\frac{1}{\rho}\frac{\partial}{\partial x_j}\frac{\rho\lambda}{(k\bar{u})^2}\,\overline{\mathrm{Re}(u_j')\mathrm{Re}(u_i')}\,\frac{\partial \bar{\mu}_0}{\partial x_i} \tag{24}$$

Thus for small λ the divergence of the eddy flux can be divided into a purely chemical and purely conservative part. The conservative part, the second term on the right-hand side of (23), is the conservative flux described by *Clark and Rogers* [1978]. The chemical part of DEF is identical to ST; therefore for small λ it can be explicitly shown that the stirring term represents the chemical contribution of the divergence of the eddy flux.

For large λ, DEF approaches zero, while ST approaches some finite value. In fact, as λ becomes large, ST loses its explicit chemistry dependence and becomes

$$\mathrm{ST} = u_i^s\,\frac{\partial \bar{\mu}_0}{\partial x_i} \tag{25}$$

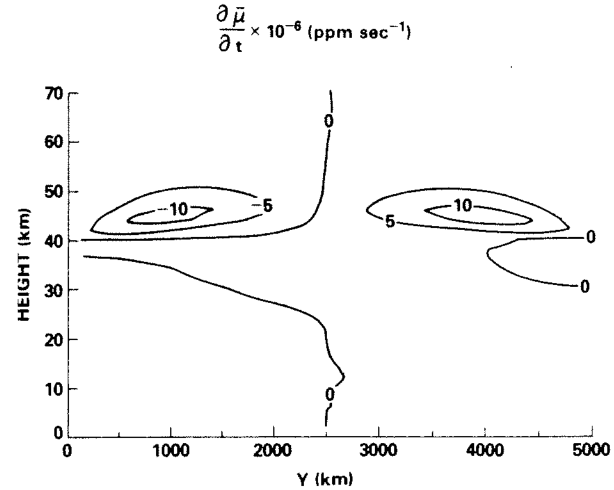

Fig. 11. Total mean ozone tendency, $\partial\bar{\mu}/\partial t$, as calculated with the Lagrangian formulation (ppm/s).

At this limit both the Eulerian mean (equation (10)) and Lagrangian mean (equation (12)) continuity equations become

$$\frac{\partial \bar{\mu}}{\partial t} = -\bar{u}_i\,\frac{\partial \bar{\mu}_0}{\partial x_i} - \lambda\bar{\mu} \tag{26}$$

Equation (26) indicates that when the photochemistry is strong, the wave effects become diminishingly small so that the mean field changes are governed by the mean chemistry and the advection by the Eulerian mean velocity.

4.5. *Lagrangian Model*

While the analysis above reveals the relationship of the stirring term to the divergence of the eddy flux, the physical nature of the stirring term remains obscured in the mathematics. *Hartmann* [1981] showed how pure oscillatory motion of a fluid parcel could produce changes in the mean ozone density in a fluid tube. A similar Lagrangian approach can be used here to investigate the effects of the planetary wave and to coalesce further the Lagrangian mean and Eulerian viewpoints.

The Lagrangian model is based on the particle trajectories as determined by the Lagrangian mean displacement fields. Using (1) and assuming that $\bar{u}$ is constant, it can be shown that for

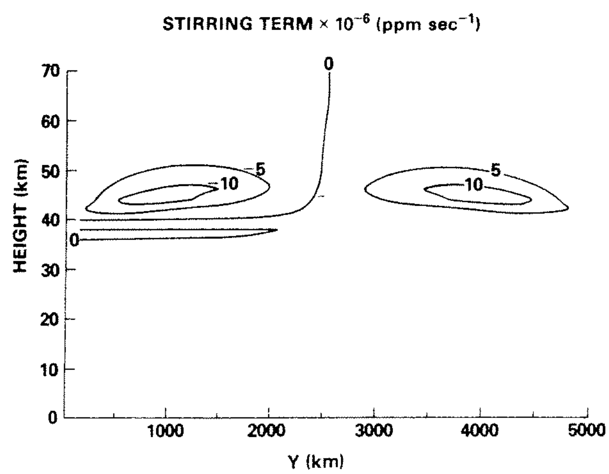

Fig. 12. Mean ozone tendency caused by 'stirring' (ppm/s).

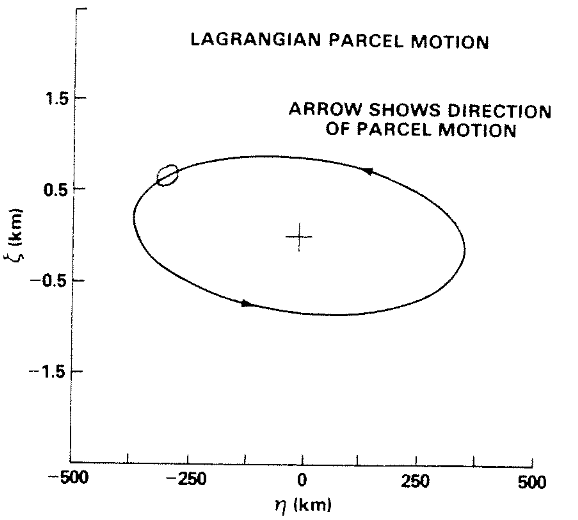

Fig. 13. Projection of displacement fields η, ζ on the y, z plane (km). For $\partial\bar{\mu}/\partial z > 0$, northward moving parcels are associated with positive perturbations.

harmonic disturbances the horizontal displacement field η and the vertical displacement field ζ are given by

$$\eta = \eta_0 \cos(\alpha z + kx) \tag{27}$$

$$\zeta = \zeta_0 \cos(\alpha z + kx) + \zeta_1 \sin(\alpha z + kx) \tag{28}$$

where α is the vertical wave number. The coefficients η_0, ζ_0, and ζ_1 are functions of height, meridional distance, and the dynamic parameters. For a particular value of y and z, the projection of η and ζ on the y, z plane is an ellipse (Figure 13). These displacement fields are similar to those calculated by *Matsuno* [1980] and *Danielsen* [1981].

The displacement fields provide a model of the parcel motion associated with a pianetary wave. The planetary wave displaces a particular parcel from its equilibrium position and causes the parcel to move in a helical trajectory around the globe. The Lagrangian mean velocity is the velocity at which the axis of the helix moves. For the small-amplitude, steady state waves of this model the vertical extent of the ellipse is of the order of 1 km and the horizontal extent is of the order of hundreds of kilometers.

First, consider a parcel in the absence of chemistry and assume that the equilibrium mixing ratio of the parcel, μ_p, is $\bar{\mu}_0$ (0, 0). If the parcel is displaced to (η, ζ), then the perturbation mixing ratio μ' is equal to the parcel mixing ratio minus the background mixing ratio. If the displacements are small, then

$$\mu' = -\eta \frac{\partial \bar{\mu}_0}{\partial y} - \zeta \frac{\partial \bar{\mu}_0}{\partial z}$$

Assuming that $\bar{\mu}_0$ is a function of z only and that it increases with height, then positive ozone perturbations are generally associated with northward motion and negative ozone perturbations are associated with southward motion (see Figure 13). This is equivalent to northward eddy flux of ozone and is a Lagrangian description of the conservative eddy flux as described by *Clark and Rogers* [1978]. This conservative flux is largely cancelled by the wave-induced mean circulation.

In the case when ζ_1 in (28) is zero, which corresponds to a trapped (nonpropagating) wave, the trajectories then become lines instead of ellipses. The perturbations are then equally correlated with both northward and southward velocity, and the conservative eddy flux is identically zero. An evanescent wave has the same property.

When chemistry is present, μ' still changes due to the varying background field, but the chemistry tries to bring the parcel's ozone mixing ratio toward the background value. The constituent density for the parcel is determined by

$$\frac{d\mu'}{dt} = S - \lambda\mu' \tag{29}$$

where S represents the effective source caused by the parcel moving through the varying background density. For elliptical trajectories, S is a harmonic function in time, proportional to the magnitude of the velocity and the background constituent gradients. In general, as the parcel orbits its equilibrium position, it moves through varying photochemisty so that λ is also a function of time. In a reference frame moving with the zonal velocity of the parcel, (29) can be rewritten as

$$\frac{d\mu'}{dt} = S_0 \cos\omega t - \lambda(t)\mu' \tag{30}$$

where $\omega = k\bar{u}$.

If λ is constant, then the solution to (30), for large t, can be written as

$$\mu' = \frac{S_0}{\lambda^2 + \omega^2}(\lambda \cos\omega t + \omega \sin\omega t) \tag{31}$$

For large λ, μ' approaches zero, which is equivalent to the previous result that the eddy fluxes disappear in the fast photochemical region. For $\lambda = 0$ the solution reduces to the ozone density appropriate to the conservative problem. Note that the average of μ' over one period is zero, and this average is equivalent to the average of the parcel density over the helical trajectory, i.e., a Lagrangian mean average.

In the regions where λ varies strongly over the extent of the ellipse the perturbation can take on a different character. Such regions would be expected to exist in the lower parts of the transition layer and at the polar night boundary where the chemistry is nearly discontinuous. Figure 14 shows an extreme example where the chemistry is discontinuous across an ellipse. As the parcel travels through the conservative region, the perturbation density varies as S. In the transition region ($\lambda/k\bar{u} = 5$)

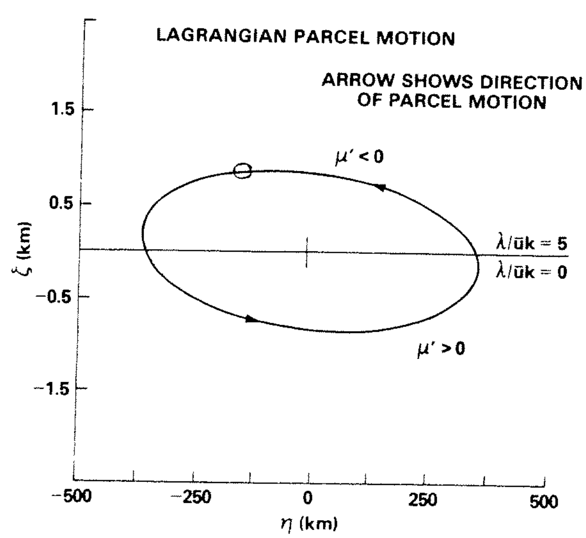

Fig. 14. Parcel orbiting through a region of discontinuous photochemistry ($\partial\bar{\mu}/\partial z > 0$).

the perturbation is decreased by ~80%. Assuming that the background density increases with height, this behavior means that the negative perturbations are selectively damped, while the positive perturbations are not. Therefore the average value of μ' over one period, which was zero when λ did not vary, may become nonzero in the presence of extreme chemical gradients. Such regions would be expected to have a great effect upon transport calculations. This effect is analogous to mean density changes in a fluid tube as calculated by *Hartmann* [1981].

Figure 15 demonstrates this effect for a more realistic problem. Figure 15*a* shows (31) the solution at large t to (30). This is a harmonic function with zero average. Figure 15*b* shows the numerical solution to (26) for an ellipse situated at the bottom of the transition layer with the model photochemistry. The steady state solution has a nonzero time average, as expected.

4.6. *The Effects of Eddy Transport on the Mean Field*

Since the changes in the mean field have been assumed to be $O(a^2)$, the calculation of the steady state forcing by the eddies on the mean field tendency is independent of the mean constituent density. Therefore at any point in space it is possible to write the mean field equation (10) or (12) as

$$\partial\bar{\mu}/\partial t = -\lambda\bar{\mu} + F \tag{32}$$

where F is the total forcing which represents the divergence of both the horizontal and vertical fluxes. If F is assumed independent of time, the solution to (32) can be written as

$$\bar{\mu} = \frac{F}{\lambda}(1 - e^{-\lambda t}) \tag{33}$$

where it has been assumed that $\bar{\mu}$, the change in the equilibrium field, is zero at time zero. As $t \to \infty$ the change in the mean field is F/λ. In the conservative region ($\lambda = 0$) the solution to (32) can be written as

$$\bar{\mu} = Ft \tag{34}$$

In order to investigate the effect of the steady state waves on the mean field, (32) was integrated for 90 days. This calculation would represent the effects of the waves if they persisted for the entire winter. The forcing is taken from the line in Figure 9 through the region of maximum ozone increase in the northern region of the domain.

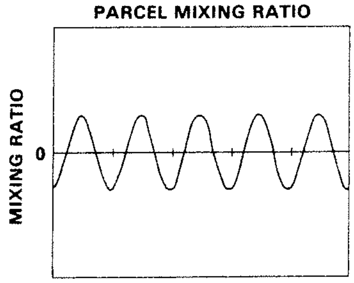

Fig. 15*a*

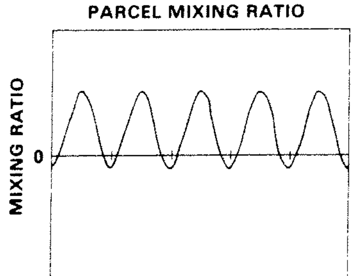

Fig. 15*b*

Fig. 15. Steady state solution of perturbation ozone density for an elliptically orbiting parcel. (*a*) Constant photochemistry. (*b*) Highly varying photochemistry at the bottom of the transition layer.

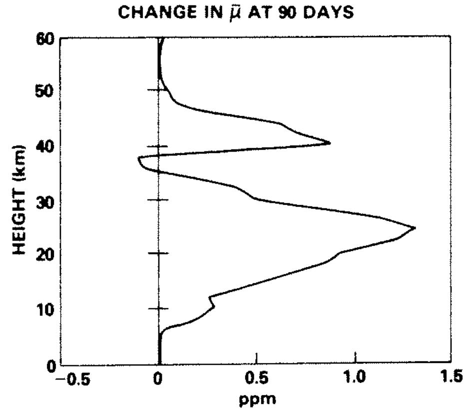

Fig. 16. The 90-day zonal mean ozone change, with the mean chemistry included, at the latitude of maximum ozone increase (ppm).

Figure 16 shows the change in $\bar{\mu}$. The large rates of change that are calculated in the transition region are almost completely counterbalanced by the mean chemistry. At the altitude of maximum increase in Figure 9 the actual change in the mean field is only 0.5 ppm for 90 days. The positive changes in the mean field at the bottom of the transition layer are in the region where the photochemistry gradients are large. This is the region where a parcel takes on a nonzero average during a parcel orbit as described in section 4.5. By considering (12) with the stirring term expanded, the Lagrangian model can be clearly related to the conventional field equations. Equation (12) can be rewritten as

$$\frac{\partial\bar{\mu}}{\partial t} = -u_i{}^L\frac{\partial\bar{\mu}_0}{\partial x_i} - \lambda(\bar{\mu} + \mu^s) - \overline{\xi_i\mu'}\frac{\partial\lambda}{\partial x_i} \tag{35}$$

If the advective terms are ignored and there are no gradients in the photochemistry and remembering that $\bar{\mu}$ is the change in the mean field, then (35) indicates that long-term changes in the mean field approach $-\mu^s$. If this term were dominant, then the largest changes in the mean field would be seen at 44 km. However, as evidenced in Figure 16, the largest changes in the region where λ is nonzero are around 40 km and are associated with the photochemical gradient term. This change behaves according to (33) and reaches its maximum value in a manner of days. The advection around 40 km tends to reduce the mean ozone (see Figure 17*a*); therefore the term dependent on the photochemical gradient is clearly dominant in the lower part of the transition region.

The largest changes in the mean field (Figure 16) are in the lower stratosphere, where changes of 1.4 ppm are seen at 90 days. These changes are described by (34) and are a linear function of time. The calculated changes in the lower stratosphere are comparable to those observed in the northern hemisphere, but immediately below the transition region is an area of depletion caused by the advection. This vertical structure of

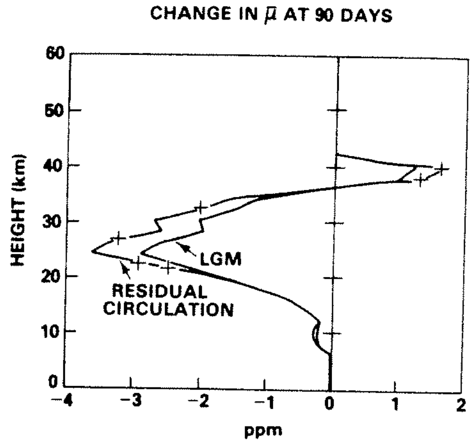

Fig. 17*a*

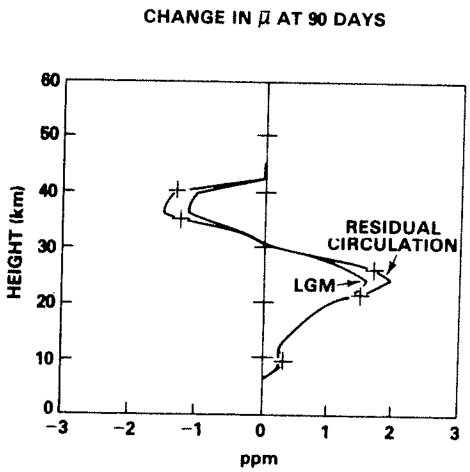

Fig. 17*b*

Fig. 17. The 90-day zonal mean ozone change, with the mean chemistry included, at northern and southern extremes, as determined by Lagrangian mean advection (LGM) and residual velocity advection (crosses) (ppm). (*a*) Southern. (*b*) Northern.

the changes in the mean field is due largely to the simplified representation of photochemistry and dynamics below the transition region.

Since the advection term is responsible for all of the change in the mean field below the transition region, it is worthy of further investigation. Mean field tendencies for 90-day periods were calculated 400 km from both the northern and southern boundaries using both the residual and Lagrangian mean advection (Figure 17). The large changes in the mid-latitudes caused by meridional transport are largely negated by the photochemistry.

In the southern region the change in the mean field by advection is unlike that observed in the stratosphere. There is a depletion of nearly 4 ppm at 24 km and an enhancement of almost 2 ppm at 40 km. In the northern region the changes in the lower stratosphere are not unreasonable, showing a 1.5 ppm increase at 24 km. However, at 36 km a 1-ppm depletion occurs. It is obvious that the modeling of the lower stratosphere is inadequate. Small-scale dynamics, photochemistry, and the radiatively driven circulation, processes which may act to counter the tendencies shown here, have all been neglected. The advection calculated using the residual circulation produces larger changes in the mean field than that calculated using the Lagrangian mean velocity. This means that in this model the antisymmetric part of *Holton*'s [1981] diffusion tensor is not negligible.

5. CONCLUSIONS

The changes in zonal mean ozone as forced by steady state planetary waves have been calculated using both a Lagrangian mean and Eulerian formulation. Although the calculated results from the two methods are the same, the use of the two methods allow for increased interpretation of the results.

In the Eulerian formulation the tendency for the eddy and mean fluxes to cancel is well illustrated. Both the horizontal and vertical flux fields are complex and largest in the conservative region. However, the changes in the mean field in the absence of the mean photochemistry are largest in the transition region, as found by *Hartmann and Garcia* [1979].

The problem of compensation by two large terms which appears in the Eulerian framework also appears in the Lagrangian mean framework in the near cancellation of the Stokes drifts and the mean velocity fields. If the Lagrangian mean flow could be calculated directly, then it might be possible to avoid the numerical difficulties of compensation; however, the Lagrangian mean quantities can be easily calculated only for simple flows. There seems to be no computational advantage of the Lagrangian mean method to calculate planetary wave transport, as it requires the formulation of Eulerian quantities, their conversion to Lagrangian mean quantities, and then the calculation of the transport.

In the Lagrangian mean formulation the stirring term is dominant in the absence of mean chemistry. The stirring is equal to the chemical contribution of the divergence of the eddy flux, when $\lambda/\bar{u}k < 1$. When λ is large the stirring term is equivalent to the Stokes advection. Using particle displacement fields, it is possible to predict the type of eddy transport that is to be expected in a certain region and it was found that μ' has a nonzero time average in the presence of strong chemical gradients. Calculation of the mean field changes shows that the mean chemistry is able to largely compensate for the eddy chemistry changes except where the chemical gradients are large. This indicates that the interaction of steady planetary waves and photochemistry need only be modeled in the presence of large chemical gradients and where $\lambda/\bar{u}k$ is relatively small.

For seasonal integrations the largest changes in the mean field are caused by advection by the Lagrangian mean velocity in the conservative region. The advection is so weak that in the chemical regions its effects are negligible. Using the residual circulation instead of the Lagrangian mean circulation overestimates the advection by the planetary wave. The largest differences between the Lagrangian mean and residual circulations occur in chemically active regions; hence, the difference between the ozone field changes calculated with the Lagrangian mean and residual circulations are small. This would not necessarily be true for constituents with different chemistry.

The transport mechanisms investigated here are probably not adequate to model planetary wave effects in the stratosphere. The use of time independent forcing without changing the mean advective field is not strictly correct. Sudden warmings, which are highly transient, will certainly have a great effect upon the ozone distribution [*Garcia and Hartmann*, 1980]. Therefore steady state planetary wave transport models are probably only relevant in the late fall and early winter. The planetary wave transport would have to be supplemented by transport due to the diabatic circulation and small-scale dynamics in the lower stratosphere. Transport by planetary waves during sudden warmings, transport by the diabatic circulation, and the relative importance of the two circulations will be presented in a subsequent paper.

NOTATION

a Newtonian cooling and Rayleigh friction coefficients.
f Coriolis parameter (1×10^{-4} s^{-1}).
H scale height (7 km).
k zonal wave number (6.28×10^{-9} cm^{-1}).
N^2 Brunt-Väisälä frequency squared (4×10^{-4} s^{-2}).
p pressure, p_0 reference pressure.
$\bar{q}_y$ horizontal derivative of the mean potential vorticity.
Q photochemical source term.
$(u, v, w) = (u_1, u_2, u_3)$, (zonal, meridional, vertical) velocity.
(v^*, w^*) (meridional, vertical) residual velocity [*Andrews and McIntyre*, 1976].
t time.
$(x, y, z) = (x_1, x_2, x_3)$, (east-west, north-south, $-H \ln p/p_0$) coordinates.
β meridional variation of Coriolis parameter f.
λ photochemical time constant.
μ ozone mixing ratio.
$(\xi, \eta, \zeta) = (\xi_1, \xi_2, \xi_3)$, (longitudinal, meridional, vertical) displacement fields.
ϕ geopotential.

Conventions, for a given Υ

$\bar{\Upsilon} \equiv$ zonal average around latitude circles (Eulerian).
$\Upsilon' = \Upsilon - \bar{\Upsilon} \equiv$ zonal deviation.
$\Upsilon^L \equiv$ Lagrangian mean [*Andrews and McIntyre*, 1978].
$\Upsilon^s = \Upsilon^L - \bar{\Upsilon} \equiv$ Stokes correction (see equation (8)).

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